POWER LAW DISTRIBUTIONS IN TWO COMMUNITY CURRENCIES

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The purpose of this paper is to highlight certain newly discovered social phenomena that accord with Zipf's law, in addition to the famous natural and social phenomena including word frequencies, earthquake magnitude, city size, income[1] *etc.* that are already known to follow it. These phenomena have recently been discovered within the transaction amount (payments or receipts) distributions within two different Community Currencies (CC) that had been initiated as social experiments. One is a local CC circulating in a specific geographical area, such as a town. The other is a virtual CC used among members who belong to a certain community of interest (COI) on the Internet. We conducted two empirical studies to estimate the economic vitalization effects they had on their respective local economies. The results we found were that the amount of transactions (payments and receipts) of the two CCs was distributed according to a power-law distribution with a unity rank exponent. In addition, we found differences between the two CCs with regard to the shapes of their distribution over a low-transaction range. The result may originate from the difference in methods of issuing CCs or in the magnitudes of the minimum-value unit; however, this result calls for further investigation.

Keywords: Power law; Zipf's law; Community Currencies; Social Experiment.

1. Introduction

After the full liberalization of capital movements in the early 1980s, free capital flows have generated new risks of instability. Because of this situation, Community Currencies (CCs)[a] have spread over the world, particularly in Western countries since the 1980s-the second CC boom after the first one in the 1930s. However, little research and few studies have been carried out to quantify how much these CCs vitalize local economies. Since the two communities of the CC cooperated, we were able to conduct two empirical studies in which we quantitatively estimated the economic effects of those two CCs. We discovered that Zipf's law applies to the distribution of the transaction amounts within those CCs; the results indicate that economic transactions create Zipf's law[2–4] without considering the type of CCs, because it is well known that income and wealth distribution with legal tender (*e.g.*, dollars or yen) follow Zipf's law. Section 2 deals with two CCs. Section 3 explains Zipf's law. Finally, we briefly comment on future research in this area.

2. Two Community Currencies

We conducted two empirical studies to estimate the economic vitalization effects of two different CCs on local economies. One of these CCs is a local one that circulates in a specific geographical area, and the other is a virtual CC that is used between members who belong to a certain community of interest (COI) on the Internet. The first CC is called Tomamae-cho ("town" in Japanese) Chiiki-Tuka ("Community Currency")[b]. Tomamae-cho is a small town in the northern part of Hokkaido in Japan with a population of about 4300 people. Its main industries are fishing, forestry, and commerce. Tomamae-cho has recently been faced with problems such as scarcity of job opportunities and spillover of purchasing power to relatively large neighboring towns (with population of 8000), in addition to long-term depopulation and aging, as in other rural areas in Japan. In 2004, the Tomamae-cho Commerce and Industry Association decided on an experimental introduction of a CC for the purpose of both economic activation and community revitalization, in collaboration with the Tomamae-cho Municipal Government.

The Tomamae-cho CC, called "P," has a linear equivalence to "yen," and the circulation works in two ways: (1) units are issued by the Commerce

[a]Community Currencies are also called Local Currencies or Complementary Currencies, with slightly different connotations. For example, Local Currencies often means the CCs that circulate in confined local areas such as towns or villages. However, we will use only the term Community Currency to avoid any confusion in terminology.
[b]This is legally a Gift Certificate redeemable into yen, so that shoppers may use it again to others or refund it with money when they receive it from their customers.

and Industry Association and the Municipal Government of Tomamae-cho when participants buy CC with yen[c], (2) participants get paid with CC for the service or goods they supply to others.

We have participated in the CC experiment and investigated its circulation flow in a unique way. Before the experimental circulation in 2004, our investigation team requested the issuing agents to devise an entry space on the back of each CC note for its recipients to record transaction dates, their names and addresses, and the purposes of use, for a maximum of five recipients. We also asked the participants to agree to cooperate in our investigation. After the circulation experiment of the Tomamae-cho CC, we collected all the currency notes that had returned to the issuing agents in exchange for yen, and we then created the adjacent matrix from the acquired data to analyze the circulation flow of the notes of the CC by using a social network theory.

The virtual CC is called LETS-Q[d]. Q is the "monetary" unit of LETS-Q; although it is not exchangeable with yen, it is equivalent to yen. The CC system was initiated in Japan in November 2001 and is managed by an administrative committee called Q-hive, comprising individuals such as professors, teachers, farmers, students, artists, and musicians. One of the authors, Makoto Nishibe, is the initiator of LETS-Q and an ex-representative of Q-hive. Coffee shops and bakeries are also members of LETS-Q apart from individual members, and are scattered all over the world. Some of them are in Tokyo and Osaka, and others in Hokkaido and even in New York. The other author, Nozomi Kichiji, is also a member of LETS-Q.

The characteristic feature of LETS-Q lies in the realization of the same type of multilateral settlement of accounts in the Internet as in the case of usual LETS systems[e]. Using this technique, participants are relieved of the constraints of geographically localized communities such that they can involve in LETS-Q irrespective of their location. Therefore, we call this type of online LETS "open communities" or "glocal" currency. Since LETS-Q was implemented on the Internet, and all related data are stored in the

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[c]500 'Yen' can buy 500 'P' note and 10 'P' stamps as its 2 percent premium.

[d]LETS stands for a Local Exchange Trading System. This system embodies the most popular type of Community Currency; it was initiated in 1983 by Michael Linton in Comox Valley, Vancouver Island, Canada, and is now utilised in more than 2000 districts worldwide.

[e]All participants in LETS start from zero in their account and cumulatively record plus and minus transactions within their accounts when they offer and receive goods or services from other participants. It is a characteristic feature of the LETS system that the summation of all accounts remains constantly at zero.

LETS-Q server, we have been able, with the help of Q-hive, to access the LETS-Q data; these data contain only the ID numbers of participants, the transaction amount, and the time stamps of settlement on the web. It does not include personal data such as name, address, and age. Using this data, we can analyze economic phenomena at the meso level of a small-sized group.

3. Zipf's Law

Zipf's law, named after the Harvard linguistic professor George Kingsley Zipf, is the observation that frequency of occurrence of a certain event(P), as a function of the rank (i) when the rank is determined by the above frequency of occurrence, is a power-law function $P_i \sim \frac{1}{i^a}$ with the exponent close to unity (1). It is widely known that Zipf's law applies to city size, income, word frequency, and earthquake magnitude. However, the mechanism of Zipf's law has not been clarified enough; accordingly, it is desirable to find more events to which Zipf's law applies. In this paper, we have presented four such events. We examined the distribution of payments and receipts of Tomamae-cho CC and LETS-Q over a certain period. The results are illustrated in Fig. 1 and Fig. 2. All figures show the rank-size relationships of the transaction amounts, the x axis is the rank, and y axis is the size. These relationships are approximately linear on a log-log plot; in other words, the ranked slopes in both the cases are nearly -1, which makes them Zipf. Thus, we discovered Zipf's law distributions in the transaction amounts within the two different CCs (local and virtual). These results indicate that transaction amounts follow Zipf's law irrespective of the type of CCs.

However, the slope of the line of best fit of the Tomamae-cho CC (-1.13) is steeper than that of LETS-Q (-1.00). This indicates that the LETS-Q is slightly more level and even than the Tomamae-cho CC in terms of transaction amounts. Moreover, the ways the two graphs fit to linear lines, as is clear from the comparison between Fig. 1 and Fig. 2, are quite different. The distribution of transaction amounts of the Tomamae-cho CC almost perfectly fits a straight line, but that of the LETS-Q deviates from it after rank 100. It can be seen that the overall Gini coefficients[f] of the Tomamae-

[f]The Gini coefficient is a measure of inequality of a distribution, defined as the ratio of area between the Lorenz curve of the distribution and the curve of the uniform distribution, to the area under the uniform distribution.

cho CC are smaller than those of the LETS-Q[g]. However, when calculating the Gini coefficients for LETS-Q for the range of rank 1 to 100, we obtain results smaller than those for Tomamae-cho CC[h]. These results indicate that the Tomamae-cho CC is more equal than the LETS-Q on the whole but LETS-Q is more equal than the Tomamae-cho CC under rank 100; this twist is caused by the aforementioned deviation of the distribution of transaction amounts with LETS-Q from the line of best fit over a low-transaction range.

The subject of concern is the reason for such a deviation in the case of LETS-Q. There are two possible explanations for this. The first is due to the differing rules under which the CCs are issued. The Tomamae-cho CC can be bought and redeemed with yen, although the same is neither permitted nor possible for LETS-Q. This creates the difference of constraints on the boundary conditions of the two CCs. The other reason is more technical; in the Tomamae-cho CC, the minimum value unit is 500 P, because only 500 P notes are issued, but for LETS-Q it is just 1Q, because transaction amounts with arbitrary numbers can be recorded in the participants' accounts. This seems to make the lower-right tail of the LETS-Q longer than that of the Tomamae-cho CC. It would appear that the two types of fit of the distributions to Zipf's law reflect the differences between the inner mechanisms and the dynamics of these two CC systems.

4. Summary

It is widely recognized that Zipf's law applies to income and wealth distribution of individuals or corporations with legal tenders. In this paper, it has been shown that Zipf's law applies to different kinds of socio-economic phenomena, such as transactions in CCs. The results suggest that a generic economic transaction can create Zipf's law, regardless of the type of currency. Moreover, we also refer to the difference with regard to how well the distributions fit Zipf's law, signifying the difference of inner mechanisms and dynamics of the two CC systems. We observe that the distributions of transaction amounts of both CCs evolve over time, but further study is required on the relation between the evolution of distributions and Zipf's law.

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[g]The overall Gini coefficients for receipts/payments with Tomamae-cho CC are 0.757641/0.719131, and those for LETS-Q are 0.772328/0.735642, respectively.

[h]The Gini coefficients for receipts/payments with LETS-Q for the range of ranks 1-100 are 0.62885/0.558051.

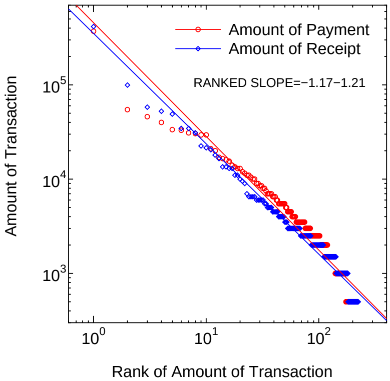

Fig. 1. Individual Rank by its transaction amount in Tomamae-cho CC. Rank of amount of transactions and the amount of transactions are plotted in the log-log scale. The plotted data are for Nov.2004-Feb.2005. The total amount of transactions is 1,385,500 P. The ranked slopes are from -1.17 to -1.21.

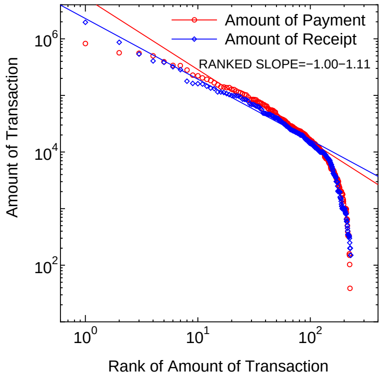

Fig. 2. Individual Rank by its transaction amount in LETS-Q. The rank of amount of transactions and the amount of transactions are plotted in the log-log scale. The plotted data are for Dec.2001-Oct.2002. The total amount of transactions is 10,220,305Q. The ranked slopes are from -1.00 to -1.11.

Acknowledgments

We like to express our appreciation to LETS-Q, Q-hive, Tomamae-cho Commerce and Industry Association and Tomamae-cho Municipal Government for their support and cooperation in our research; and to Takayoshi Kusago, Ippei Hozumi, Kenichi Kurita, Kenichi Yamamoto, Masayuki Yoshida, and Satoshi Yoshii for their collaborations in Tomamae-cho CC experiments. We also thank Satoshi Tanda and Victor M. Yakovenko for their advice and comments. This work was supported by Hokkaido University's 21st century COE program "Topological Science and Technology."

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