Evolut Inst Econ Rev (2017) 14:1–27
DOI 10.1007/s40844-017-0071-8

ARTICLE

Theoretical model of institutional ecosystems and its economic implications

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Published online: 3 May 2017

Abstract Previous game-theoretic studies of institutions have viewed institutional changes as either exogenous changes in game form or changes in the game equilibrium through exogenous shocks. Both views of institutions are static and cannot express endogenous changes in institutions. The latter approach states that multiple institutional systems can be kept stable through institutional complementarity and that the changes in institutional systems only arise from exogenous shocks that are sufficiently large to overturn such complementarity. However, they cannot account for the aspects of competition and co-existence where multiple institutions change their relative frequency through endogenous changes. In this article, we model the ecological systems of institutions, as an extensive synthesis of replicator dynamics and evolutionary games, to describe institutional systems that evolve phylogenetically associated with changes in population structure or a pool of rules as replicators, which corresponds to a gene pool. A mathematical model of rule ecosystem dynamics describes rule dynamics wherein multiple rules change their relative weights through evaluations by individuals. In this model, the concept "a meta-rule = an individual value consciousness" is introduced for the rule

This article is the revised English version of the Japanese original version (Hashimoto and Nishibe 2012).

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evaluation. Depending on the setting of the meta-rule, the dynamics of the game rules and individual strategic rules change. We can thus comprehend the endogenous formation, alteration, and extinction (i.e., the evolution of institutions) through the interactions among the game rules as well as those between the game rules and strategic rules. Many other studies focus on how rational individuals select strategies to maximize their payoffs without considering the bounds of rationality. Even when considering these, individual cognitive frameworks and values are typically given. By contrast, this study assumes that individuals have internal rules that express cognitive frameworks and values as meta-rules and analyzes the dynamic interactions between institutions, as social external rules at the meso level, and strategic rules and value consciousness, as individual internal rules at the micro level. In our model, institutional changes do not arise as game equilibria (i.e., players' selection of strategies in a game), but rather as the rise and fall of game forms, as various rules, in multi-games based on a meta-rule. This view is based on an evolutionary approach where socio-economic evolution is considered to be a selection of rules and institutions rather than that of individuals or their strategies. We discuss the implications of the model of institutional ecosystems on the description of the socio-economy and its evolutionist institutional design.

Keywords Replicator as rule · Institutional variety and endogenous change · Institutional ecosystem · Rule ecosystem dynamics · Meta-rules · Evolutionist institutional design · Micro–meso–macro loops

JEL Classification B25 · B52 · C73 · E14 · E42

1 Introduction

Institutions, as ways of thought and behavior common in society (Veblen 1899), consciously or unconsciously regulate our thought and behavior in some way and enable our social lives. Most of these institutions are not given to us a priori, but they are rather spontaneously formed from our behavior. Institutional changes refer to the rule dynamics (Hashimoto and Nishibe 2005; Hashimoto 2006) that arise from the interactions between institutions and individual behavior regulated by these institutions.

In regard to institutions, there are two viewpoints from a game-theoretic perspective: "institutions as game forms" (North 1990; Hurwicz 1996) and "institutions as equilibria of strategies" (Lewis 1969; Pagano 1992; Pagano and Rowthorn 1994; Young 1998; Aoki 2001).

On the one hand, the former states that institutions are the rules of interactions. Here, an institution is described by a game form that consists of a set of players, options, and consequence functions (payoff matrices); individual behaviors are then described as options in the game. In this case, as long as the game form is not changed externally, there will be no change in the institution. On the other hand, the latter regards an institution as the Nash equilibria of a game and individual behaviors as the moves chosen on the basis of the strategies in the game. In addition, an established institution is characterized by the combinations of strategies at the equilibria. To make this static framework dynamic through the introduction of an evolutionary game, we can treat the establishment of institutions as a starting point for strategy combinations out of an equilibrium state, moving toward those in an equilibrium state. However, once an institution has been established, it is in an equilibrium; therefore, as long as there are no exogenous shocks to displace the state from the equilibrium, the institution will not change.

Accordingly, from both perspectives, an established institution shows no endogenous changes and it will remain static as long as no exogenous disruptions exist. In this paper, we integrate these two game-theoretic viewpoints of institutions and introduce a mathematical model that can explicitly deal with rule dynamics. Institutions are formed, maintained, altered, and eliminated through the meso-level interactions among individual cognition, thought, and behavior (micro) and social consequences (macro), and are formed by alternative or complementary relationships in those processes.

Institutions are of two types: "outer institutions", as explicitly enforced social rules such as laws, regulations and game forms, and "inner institutions", as spontaneously formed rules such as values, norms, ethics, collective consciousness, and standard tactics. We suppose that both types of institutions are composed of "if–then" rules (replicators) shared by many individuals (interactors) located at the micro level. It is then ontologically seen that such rules shared within a group or society exist at the meso level (i.e., above individuals at the micro level and below social consequences at the macro level). Accordingly, both outer and inner institutions could be called "external rules" because they act on individuals from the outside. On the contrary, individual-specific if–then rules (replicators) are called "internal rules" since they determine the cognition, thoughts, and behaviors of individuals from the inside. Those rules and institutions are arranged as follows:

Meso level: external rules = institutions (outer institutions and inner institutions).
Micro level: internal rules.

Each individual (interactor) exhibits routinized cognition, thoughts, and behaviors based on replicators as both external rules and internal rules (Hayek 1967; Nishibe 2005, 2006, 2010a).[1] Accordingly, these types of individuals are not rational entities that pursue optimality. While there are limits on their ability to

[1] The definition and distinction of external/internal rules and outer/inner institutions by treating meso level institutions as game rules (outer institutions) and meta-rules as game rule evaluations (inner institutions) are different from previous ones (Nishibe 2006). While the game rules (outer institutions) set the range of behavior for each individual "from the outside", or in a top-down manner, meta-rules (inner institutions) set the nature of the total game "from the inside" by using the relative frequency of each game rule, or in a bottom-up manner. While the "external/internal" in external/internal rules represents the static "boundaries of a set" or "areas", "outer/inner" implies a dynamic "causal direction" or "determined relationship direction." For this reason, we use the terms inner/outer institution. Note that outer and inner institutions are not distinguished by the explicit or implicit sharing of rules among individuals. If that were the case, then it would be more appropriate to use the terms "explicit/implicit institution" or "formal/informal institution." In that case, the classifications would become closer to legal and regulatory/ethical and customary. Then, "implicit" or "informal" institutions would merely be unexpressed conventions or codes that arise within the domain of freedom set by "explicit" or "formal" institutions, and eventually fill the space left by "explicit" or "formal" institutions. The problem is that if "implicit" or "informal" institutions are only implicit parts of the game rules, they cannot be meta-rules for value consciousness on the fairness and appropriateness of the game rules at the meta level; thus, it would be impossible to show mutual determination between the outer and inner institutions found in this paper. Here, meta-rules that regulate the game rules are given for a provisional treatment. However, this does not mean that they exist transcendentally from the outset. They can change dynamically and endogenously. A meta-rule is

reason, calculate, and act, they are realistic in how they adapt to actual changes. An institutional ecosystem is a type of system that continuously maintains a variety of institutions within the process of multiple institutions co-existing and becoming extinct in an unending micro–meso–macro loop (Nishibe 2005, 2006, 2010a, 2012; Hashimoto et al. 2010; Kobayashi et al. 2013). We aim to construct a mathematical model for the institutional ecosystem, named rule ecosystem dynamics (RED), which describes the dynamics of this type of system (Hashimoto and Nishibe 2005).

The model outlined in this paper is, mathematically, an extension of the replicator dynamics commonly used in evolutionary game theory. First, we treat external institutions as the interaction rules (game rules hereafter) and represent the game form (payoff matrix). Replicators (internal rules), which take the form of individual behavior, are regarded as the game strategies and called "strategy rules". Moreover, to show the results of the interactions among multiple outer institutions, we introduce multiple games along with their respective weights. From now on, weights are used to express the significance of the game rules. Each player plays all games and earns a payoff multiplied by the weight of each game. The strategy rule distribution varies with time depending on the payoff. Along with the strategy rule distribution, the game rule weights also vary with time, with the variations depending on the payoff and strategy rule distribution.

In this study, we introduce "meta-rules" anew to determine how the game rule weights change. These meta-rules are used to evaluate the game rules and are thought to be at a more basic level than the game rules when viewing changes. Such meta-rules, in general, are not necessarily expressed explicitly, but rather should be considered to be existing implicitly as a collective representation of the game rule evaluations made by game participants. In other words, these meta-rules are external rules as inner institutions. Note that this type of evaluation does not evaluate the personal payoff or utility gained as a result of the game but rather the game rules themselves. This evaluation thus differs from so-called consequentialism. The meta-rules discussed in this paper are not a type of aggregate rules set to obtain a social welfare function composed of each member's utility or the social preferences from each member's preferences to evaluate society's overall satisfaction from a utilitarian standpoint.[2] In addition, these meta-rules are exogenously given; we consider them to be neither (game) rules nor the consequences of rules.

Footnote 1 continued
synthesized as value consciousness from individuals' evaluation in their minds. During the process, it is then transferred from the internal to the external.

[2] Games that explicitly introduce bidirectional causality between the game rules and meta-rules are generally understood to be "triangular game processes" (Nishibe 2006). Meta-rules herein are of two types: "definition and modification meta-rules", which predefine the rules and modify them on the basis of ex post facto evaluations, and "standard meta-rules", which evaluate the rules on the basis of the game results. In RED introduced herein, meta-rules are first understood to be "standard meta-rules." This is the game evaluation function $\lambda^g = \lambda^g(\boldsymbol{x}, \boldsymbol{u}^g)$ given in Eq. (21). Each game rule is evaluated from high to low on the basis of this function. In general, game rule g is modified after having been evaluated low by these standard meta-rules (or perhaps by violating a meta-rule standard). For example, if a certain law or rule is determined to be unconstitutional, it will likely be altered. However, in RED, game rule g itself is not altered or edited after receiving a low evaluation, but rather its weight, w^g, will decrease. In this manner, "definition and alteration meta-rules" include situations where only the weight, w^g, of game rule g is modified; game rule g itself is not modified. Within triangular game processes, it is possible to include meta-meta-rules, even higher-level processes that can alter meta-rules. As outlined in this paper, within RED, meta-rules are assumed to be exogenous and do not include the processes wherein they are revised.

An example of a meta-rule is profit orientation in the market economy. If we say that each market is evaluated by the sum of all the market transactions within the broad principles of the market economy, then the markets that generate a greater profit to participants overall are evaluated higher and their weights increase, whereas the ones that do not will be evaluated lower and their weights decrease. Because of this, transactions in weaker markets diminish until these markets have no choice but to disappear eventually. Alternatively, norms and value consciousness, such as those related to ethical foundations in utilitarianism or egalitarianism, express meta-rules. In the former, the rules closest to producing the greatest happiness for the greatest numbers are evaluated higher, while in the latter rules that produce the fewest disparities among people are evaluated higher.

In RED as introduced herein, meta-rules do not change and are set exogenously. Accordingly, one should be empirically led toward reasonable meta-rules because we can expect it to be difficult or even impossible to discuss a priori what type of meta-rule is normatively the most desirable [as in Arrow's (1951) impossibility theorem]. In addition, meta-rules are collective representations of the norms and values formed by game participants according to the game rules and hence cannot be easily made explicit. In other words, they must be discovered by empirical research.

While each game rule (or game form) does not change, the overall rules of society, as a set of games, are modified through changes in the weights of the game rules. There are many levels of game rule plasticity (or ease of change), so we must assume that the game rules have a hierarchy. For example, let us consider the system of law. We have, in a constitutional state such as Japan, constitutional law, criminal law, civil law, regulations, and many other types of rules. Within these are easily changeable types such as regulations or civil law as well as difficult-to-change types such as criminal law. We also have constitutional law, which constrains both criminal and civil law; this is harder to change, and it sets forth the principle to change the lower-level rules. Within the hierarchy of plasticity, meta-rules are seen as more universal and less plastic compared with the rules on which we are focusing. If we are interested in changes at the civil law level, we treat the constitutional law level and its unchanging nature as given.

Both the strategy rule distribution and the game rule weight distribution change in this framework. These distributions may be in equilibria or show metastability with certain periods of steady states. The case of metastability is thought of as a state with a provisional institution.

In this paper, we first derive RED from replicator dynamics and then perform a simple simulation analysis. Next, we consider examples of the institutional ecosystem and meta-rules and discuss future directions and economic implications.

2 Introduction to RED

RED is defined as an extension of replicator dynamics. In this section, we explain replicator dynamics as the foundation, multi-game dynamics, which extend interactions into multiple games, and finally RED, wherein we introduce the dynamics of the games themselves.

2.1 Replicator dynamics

People have certain cognition, thought, and behavioral patterns in everyday life. These patterns are the "internal rules" that peoples' cognition, thoughts, and behaviors follow within customs, habits, inclinations, and values. They are different from "external rules", which constrain peoples' range of cognition, thoughts, and behaviors externally as societal laws and regulations (outer institutions) or values and norms (inner institutions). These patterns are typically expressed as sets of if–then rules. They are formed through education and traditions at home, at school, and in the community; they are then spread by imitation and studied via media or direct interaction. Such patterns of cognition, thoughts, and behaviors are propagated and spread in this manner to families, communities, schools, corporations, markets, states, and other groups of people. We consider these "patterns" to be replicators (Nishibe 2006, 2010b).

In general, the propagation dynamics of replicators can be described by replicator dynamics. The following differential equation describes the changes in the frequency of N-type replicators (as a ratio of population) in interactions as a game:

$$\dot{x}_i = x_i(u_i - \bar{u}), \quad (i = 1 \sim N), \tag{1}$$

where the variable x^i is the i-th replicator's frequency and this satisfies the following equation:

$$\sum_{i=1}^{N} x_i = 1, \tag{2}$$

the variable u_i is the payoff gained by the i-th replicator,

$$u_i = \sum_{j=1}^{N} E(i,j)x_j, \tag{3}$$

and $\bar{u}$ is the average payoff written as

$$\bar{u} = \sum_{i=1}^{N} x_i u_i = \sum_{i=1}^{N} \sum_{j=1}^{N} E(i,j)x_i x_j, \tag{4}$$

by supposing[3] $E(i, j)$ as the payoff resulting from the interactions between replicators i and j.

Here, we make the following simplification. First, each individual interacts with the others in a certain game under the condition that the outcome of the individual's behavior depends on others' behavior. In general, individuals interact with the others within various games, but here we use only one game. Second, each individual has only one internal rule for cognition, thought, and behavior within the game. Accordingly, the individual's characteristic is expressed through an internal

[3] $E(i,j)$ need not be given prior to interactions; it only needs to be the sum of the interaction results. Replicators are behavioral patterns and not thoughts to be used to select behaviors on the basis of the pre-calculation of the game results.

rule as a replicator. This individual imitates a replicator used by another well-behaved individual (i.e., a replicator highly gained in the game). Thus, while each individual is characterized by one replicator, one replicator can be used by many individuals. In this manner, the relative frequencies of successful replicators increase. Within replicator dynamics, the rate of imitation is proportional to the degree of success. More concretely, replicators with a higher-than-average payoff increase in proportion to the difference from the average, while those with a lower-than-average payoff decrease.[4]

We set no particular specifications for $E(i, j)$, the function that determines the payoff (i.e., the outcome function of the game). We do, however, assume that two individuals gain some payoff as an outcome of behaving and interacting according to their own replicators.

2.2 Multi-game dynamics

An institutional ecosystem conceptualizes social reality not as independent institutions but as the co-existence of and interactions among multiple institutions. In other words, in an institutional ecosystem, individuals simultaneously share game-theoretic interactions with other individuals in multiple games rather than playing only one game. Multi-game dynamics is thus an extension of replicator dynamics for describing an individual as playing multiple games simultaneously.

To make our explanation easy to understand, we first use the example of two games being played simultaneously and then expand that example into a general one. By representing the payoff matrices of two games by $\boldsymbol{A}$ and $\boldsymbol{B}$, where the number of possible moves (matrix rank) can be different, and considering a strategy rule as a set of replicators (a combination of the two replicators used in each game), we rewrite the strategy rule as follows:

> Strategy rule (i, j): replicator i is used in game rule $\boldsymbol{A}$ and replicator j in game rule $\boldsymbol{B}$.[5]

We suppose that x_{ij} is the frequency, which is the population share relative to the whole, of an individual with strategy rule (i, j), where $\sum_{i,j} x_{ij} = 1$, and that

[4] In general, explanations of replicator dynamics in the evolutionary game theory literature call these replicators "strategies" . Strategies are easily confused with moves in games, so some care must be taken. A strategy is a method for selecting a move. In individuals, replicators correspond to strategies and behaviors to moves. Different strategies (replicators) may make the same move (behave in the same way). The term "strategy rules" used in this paper refers to this type of strategy. For example, the tit-for-tat strategy in the iterated prisoner dilemma game is an if–then rule that states "if the other player cooperated in the last game, then I will cooperate with him/her this time; otherwise, if the other player betrayed me in the last game, then I will betray him/her this time". This is a replicator of an individual. The specific moves are "cooperate" and "betray". The All-C strategy can be written as "if *, then cooperate", where "*" is a wild card representing any behavior. In other words, no matter what the past history between another player and an individual, this strategy is to always cooperate with the others. These two strategies execute the same "move", cooperate, when the other player has cooperated in the last game.

[5] Note that this is different from a strategy profile, which is a set of strategies selected by players for one game rule and represents a state of a society. Strategy rule (i, j) herein expresses one's way of deciding behavior, namely the internal rule, for two game rules.

$E[(i,j),(k,l)]$ is the payoff gained by the individual with strategy rule (i,j) in a competition between strategy rules $(i,\ j)$ and $(k,\ l)$. Thus,

$$E[(i,\ j),(k,\ l)] = \boldsymbol{A}_{ik} + \boldsymbol{B}_{jl}, \tag{5}$$

where $\boldsymbol{A}_{ik}$ represents the element of row i and column k of matrix $\boldsymbol{A}$. We assume that the two games are independent and that the payoff from each game independently contributes to the total payoff of each player.

By using these expressions, we can now write the frequency changes (replicator dynamics) of an individual with strategy rule (i,j) as

$$\dot{x}_{ij} = x_{ij}\left\{\sum_{k,l}(\boldsymbol{A}_{ik} + \boldsymbol{B}_{jl})x_{kl} - \sum_{i',j'}\sum_{k,l} x_{i'j'}(\boldsymbol{A}_{i'k} + \boldsymbol{B}_{j'l})x_{kl}\right\}. \tag{6}$$

This is organized as follows:

$$\begin{aligned}\dot{x}_{ij} &= x_{ij}\left\{\left(\sum_{k,l}\boldsymbol{A}_{ik}x_{kl} - \sum_{i',j'}\sum_{k,l}x_{i'j'}\boldsymbol{A}_{i'k}x_{kl}\right) + \left(\sum_{k,l}\boldsymbol{B}_{jl}x_{kl} - \sum_{i',j'}\sum_{k,l}x_{i'j'}\boldsymbol{B}_{j'l}x_{kl}\right)\right\}\\ &= x_{ij}\sum_{g=1}^{2}(u_{ij}^{g} - \bar{u}^{g}),\end{aligned} \tag{7}$$

where u_{ij}^{g} is the payoff that strategy rule (i,j) gains in game rule g, namely

$$u_{ij}^{1} = \sum_{k,l}\boldsymbol{A}_{ik}x_{kl}, \quad u_{ij}^{2} = \sum_{k,l}\boldsymbol{B}_{jl}x_{kl}, \tag{8}$$

and $\bar{u}^{g}$ is the average payoff in game rule g,

$$\bar{u}^{1} = \sum_{i',j'}\sum_{k,l}x_{i'j'}\boldsymbol{A}_{i'k}x_{kl}, \quad \bar{u}^{2} = \sum_{i',j'}\sum_{k,l}x_{i'j'}\boldsymbol{B}_{j'l}x_{kl}. \tag{9}$$

As multi-game dynamics with two games can be simply written as in Eq. (7), we can obtain the generalized form of multi-game dynamics by playing M games as follows:

$$\dot{x}_{i_1\cdots i_M} = x_{i_1\cdots i_M}\sum_{g=1}^{M}(u_{i_1\cdots i_M}^{g} - \bar{u}^{g}), \tag{10}$$

where a strategy rule $(i_1,\ i_2,\cdots,i_M)$ is the internal rule or a set of replicators, “playing game rule 1 with replicator $i_1,\cdots$, game rule g with replicator $i_g,\cdots$, game rule M with replicator i_M”, and its frequency is $x_{i_1\cdots i_M}$. This also satisfies the following equation:

$$\sum_{i_1=1}^{r^1}\sum_{i_2=1}^{r^2}\cdots\sum_{i_M=1}^{r^M} x_{i_1\cdots i_M} = 1, \tag{11}$$

where r^g is the number of options within game rule g, that is, the number of replicators and the rank of payoff matrix $\boldsymbol{A}^g$. The variable $u^g_{i_1\cdots i_M}$ is the payoff of the strategy rules $(i_1, i_2, \ldots, i_M)$ in game rule g:

$$u^g_{i_1\cdots i_M} = \sum_{k_1\cdots k_M} \boldsymbol{A}^g_{i_g k_g} x_{k_1\cdots k_M}, \tag{12}$$

and the variable $\bar{u}^g$ is the average payoff in game rule g:

$$\bar{u}^g = \sum_{j_1\cdots j_M}\sum_{k_1\cdots k_M} x_{j_1\cdots j_M} \boldsymbol{A}^g_{j_g k_g} x_{k_1\cdots k_M}. \tag{13}$$

2.3 RED

Researchers have analyzed institutions by using the multi-game framework that illustrates individual social behavior within multiple institutions (Gagen 2000, 2003; Sallach et al. 2010). How such a multi-game framework changes the equilibrium structures and dynamics of elemental games has also been analyzed (Hashimoto 2009; Hashimoto and Aihara 2009). Within the multi-game dynamics introduced in this paper, individuals are expressed as the strategy rules (internal rules), that is, sets of replicators, playing multiple games. From the standpoint of the game rules (game forms) as institutions, this framework illustrates the replicator dynamics in multiple institutions where the populations of the strategy rules (replicators) that have higher-than-average payoffs increase their relative frequencies, while those that do not decline. However, the conception so far, namely, multi-game dynamics, still considers institutions to be static.

As stated above, institutions are not static entities within the institutional ecosystem. At the meso level, they intermediate between individual cognition, thought, and behavior at the micro level and societal consequences at the macro level, interacting with these two levels. Through these interactions, institutions emerge, alter, and diminish; further, an institutional ecosystem with substitutive and complementary relationships among institutions changes over time. This is the dynamics of the micro–meso–macro-loop.

Let us now extend the multi-game dynamics to express these dynamics. We consider that each game rule in the multiple games has its weight that changes over time with the time development of the strategy rule distribution. By introducing weight w^g of game rule g, we rewrite Eq. (10) as

$$\dot{x}_{i_1\cdots i_M} = x_{i_1\cdots i_M}\sum_{g=1}^{M} w^g\left(u^g_{i_1\cdots i_M} - \bar{u}^g\right). \tag{14}$$

This represents the dynamics of the strategy rule distribution.

The weights of the game rules also have dynamics, which are assumed to depend on how individuals evaluate each game rule (outer institutions of external rules) including the payoff structure (or payoff matrix). Game rules with higher-than-average evaluations increase their weights, while those with lower-than-average evaluations decrease their weights. In other words, the weights of the game rules change just as in replicator dynamics according to the evaluations of the game rules. The dynamics can be written as

$$\tau \dot{w}^g = w^g(\lambda^g - \bar{\lambda}), \tag{15}$$

where λ^g is the evaluation of game rule g, and

$$\bar{\lambda} = \sum_{g=1}^{M} w^g \lambda^g, \tag{16}$$

is the weighted average of the evaluations. The parameter τ, a time constant, determines the rate of change of the game rules (i.e., the ratio of the changing speed of all institutions to that of individual strategies). The weight of game rule g satisfies the following equation:

$$\sum_{g=1}^{M} w^g = 1. \tag{17}$$

Now, the problem is how to evaluate the game rules. We believe that they can be evaluated by using the meta-rules discussed above. A meta-rule is the collective representation of individuals' value consciousness to evaluate the game rules and is, in RED, given as an evaluation function for the game rules. The evaluations of game form (external rule) arise as a result of the cognition, thought, and behavior according to the particular strategy rules (internal rules or replicators) within games. Accordingly, the evaluation of the game rules must be a function of the strategy rule profile and the payoff profile. In other words, they take the following form:

$$\lambda^g = \lambda^g(\boldsymbol{x}, \boldsymbol{u}^g), \tag{18}$$

where $\boldsymbol{x} = (x_{1,1,\ldots,1}, x_{1,1,\ldots,2}, \ldots, x_{r^1,r^2,\ldots,r^M})$ is a strategy rule profile (a vector of the population ratio of each strategy) and $\boldsymbol{u}^g(\boldsymbol{x}) = (u^g_{1,1,\ldots,1}(\boldsymbol{x}), u^g_{1,1,\ldots,2}(\boldsymbol{x}), \ldots, u^g_{r^1,r^2,\ldots,r^M}(\boldsymbol{x}))$ is a payoff profile (a payoff vector for each strategy rule) within game g.

In summary, RED is defined by the following three equations:

$$\dot{x}_{i_1 \cdots i_M} = x_{i_1 \cdots i_M} \sum_{g=1}^{M} w^g (u^g_{i_1 \cdots i_M} - \bar{u}^g), \tag{19}$$

$$\tau \dot{w}^g = w^g(\lambda^g - \bar{\lambda}), \tag{20}$$

$$\lambda^g = \lambda^g(\boldsymbol{x}, \boldsymbol{u}^g). \tag{21}$$

Equations (19)–(21) represent the changes in individual replicators (strategy rule distribution), the dynamics of the weights of the game rules, and the evaluation of the games, respectively. Table 1 presents the characteristics of this framework, in comparison with those of replicator dynamics and multi-game dynamics, as shown in Table 1.

In addition to replicators (or behavioral patterns), as with replicator dynamics, that express the internal rules of individuals, or the strategy rules used by individuals, we consider, in RED, the game rules (external rules) as a kind of replicator in the multi-game framework and introduce their selection dynamics. The selection dynamics of the two types of replicators, micro-level strategy rules (internal rules) and meso-level game rules (an external rule as an outer institution), can be seen in Eqs. (19) and (20), respectively. Each type exists simultaneously at different levels. Individuals are seen as mere vehicles (interactors) for these two categories of replicators. The meta-rule in Eq. (21) determines the rise and fall of the game rule replicators. Although we say that individuals are vehicles for these two types of replicators, they are collectively engaged in forming the game evaluation functions in Eq. (21) through value consciousness.

3 Examples of RED

We now discuss two examples, kept as simple as possible, of the evaluation functions of the game rules and meta-rules and show their dynamics by using simulations in order to demonstrate how they actually act.

3.1 Examples of meta-rules

One example is the average-payoff-type meta-rule defined by

Table 1 Comparison of the three frameworks

	Replicator dynamics	Multi-game dynamics	Rule ecosystem dynamics
Individual characterization	Replicators (strategies)	Replicator groups (strategy rules)	Replicator groups (strategy rules)
Games played	One game	Multiple games	Multiple games
Game rules	No weights	No weights	Weighted
Payoffs	No weights	No weights	Weighted
Game evaluations	None	None	According to meta-rules
Meta-rules	None	None	Distribution of strategy rules and scores

$$\lambda_{\mathrm{A}}^{g}(\boldsymbol{x},\boldsymbol{u}) = \sum_{i_1 \cdots i_M} x_{i_1 \cdots i_M} u_{i_1 \cdots i_M}^{g}. \tag{22}$$

The higher the average payoff gained from the game rules, the higher those rules are evaluated. Note that $\lambda_{\mathrm{A}}^{g}(\boldsymbol{x},\boldsymbol{u})$ is not the total of each individual's payoff or each strategy rule's payoff, but rather the total payoff of the strategy rules for each game rule. For example, among diversified exchanges, which have various regulations and provide various financial products, we can assume a meta-rule in financial markets that regards exchanges giving a large profit on average highly. This is similar to the case that the total of each individual's utility functions represents social welfare under the supposition of cardinal utility.

Another example is the inverse-variance-type meta-rule defined as follows:

$$\lambda_{\mathrm{IV}}^{g}(\boldsymbol{x},\boldsymbol{u}) = \sum_{i_1 \cdots i_M} \left\{ x_{i_1 \cdots i_M} \left(u_{i_1 \cdots i_M}^{g} - \bar{u}^{g}\right)^2 \right\}^{-1}. \tag{23}$$

This is an egalitarian meta-rule, namely a game rule with a larger variance in individuals' payoffs that is lower evaluated. This corresponds, for example, to a social democratic meta-rule in which it is thought that a government should correct economic disparity by income redistribution.

3.2 Simplifications and simulation settings

The two types of replicators, strategy rules and game rules, make RED a high-dimensional dynamical system. To show the basic behavior of RED by using the two meta-rules introduced above, we simplify the system by posing several constraints. First, we set the number of options of all the game rules to be the same, expressed herein as N:

$$r^{g} = N, \quad (g = 1 \sim M). \tag{24}$$

This means that the types of thoughts and behaviors that can be selected are the same in number in a situation of interactions (i.e., in all the elemental game rules being played simultaneously). We also limit the strategy rules to select an identical strategy number (strategies with the same row and column) for all the game rules:

$$x_{i_1 \cdots i_M} = x_i. \tag{25}$$

Individuals that select strategy rule 1 for game rule 1 will choose strategy rule 1 even for game rule 2, and so on. Because the numbering of the strategy rules is arbitrary and the game structure does not change under the simultaneous permutation of rows and columns in the matrix, the generality of strategy rule numbers does not decrease.[6] There is, however, a constraint that individuals who choose the same strategy rule for a game rule must choose the same strategy rule for

[6] When an individual choose strategy rule 1 for game rule 1 and strategy rule 2 for game rule 2, replacing row 1 with 2 and column 1 with 2 in the game rule 2 matrix makes the individual select strategy rule 1 for game rule 2.

the other game rules. In other words, individuals who choose "strategy rule 1 for game rule 1 and strategy rule 1 for game rule 2" cannot co-exist with those who choose "strategy rule 1 for game rule 1 and strategy rule 2 for game rule 2". This constraint means that all the strategy rules (internal rules) of an individual should be consistent with each other and with his/her customs, habits, routines, norms, and values. For example, although a corporation may adjust its strategy rules depending on each market, its core strategy rule remains the same; one's personality is not so free to behave differently depending on the differing game rules, but its basic personality is nevertheless the same. Mathematically, this limits the possible diversity of replicators to a subspace, but may be useful as the first steps toward understanding the nature of RED.

3.3 Simulation results

We determine the number of game rules M and strategy rules N for each game. In the following simulations, the elemental game rules are randomly generated with the average payoff 0 and variance 1. The time constant $\tau = 1000$. The initial shares are the same for all the strategy rules.

With these settings, we first show a typical example of the average-payoff-type meta-rule (Eq. (22)). Figure 1 shows a typical example of the changes over time for the population shares of the strategy rules (a) and weight shares of the game rules (b) for the average-payoff-type meta-rule. In this case, one particular strategy rule rapidly dominates. Following the domination, a game rule in which the dominant strategy rule earns the highest payoff grows the most to obtain the highest weight share. This phenomenon is a kind of globalization.

When a particular strategy rule becomes successful to gain population share, the game rules in which that strategy rule gains more become highly evaluated and regarded as being relatively important. Then, the frequency of the successful strategy rule that succeeds in the game rules increases again. They form a positive feedback loop, and finally the strategy rule and game rule, respectively, monopolize. The rate of following the strategy rule monopolization by the game rule monopolization depends on the parameter τ to decide the time constant. The phenomenon of monopolization by both a strategy rule and a game rule, however, always occurs with the average-payoff-type meta-rule.

Figure 2 shows an example of the inverse-variance-type meta-rule, with $N = 3$ and $M = 10$.[7] While there are periods in which a strategy rule dominates, those periods do not last long, and the dominant strategy rules are continually replaced in stark contrast to the case of the average-payoff-type meta-rule. Likewise, there are periods in which a game rule dominates, but the dominant game rules are continually changing place as well. This phenomenon of continual replacement is everlasting; while the period of replacement becomes long, as a saddle network, the characteristic of replicator dynamics is formed. The sudden changing of the dominant game rules, such as in revolutionary changes in social structure, is a feature of this meta-rule.

[7] The other settings for the simulation are the same as in Fig. 1.

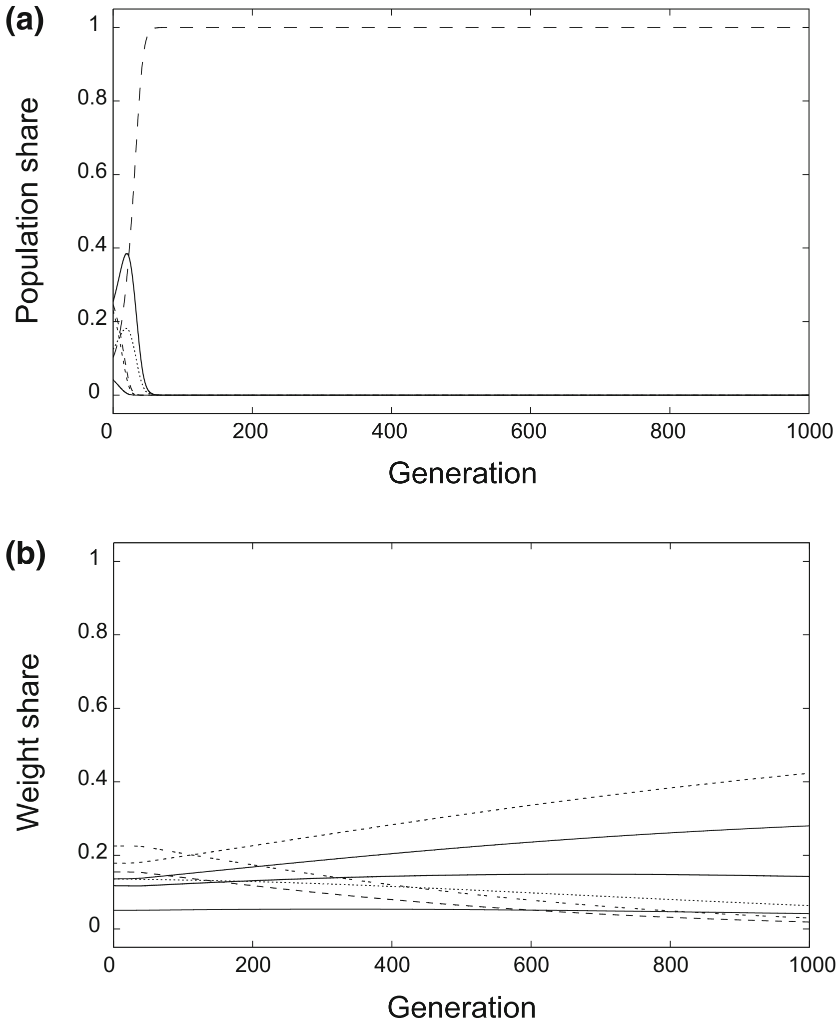

Fig. 1 Typical example of the dynamics of the strategy rules (**a**) and game rules (**b**) within the average-payoff-type meta-rule (six strategy rules, seven game rules, $\tau = 1000$)

In the case of inverse-variance-type meta-rules, when a particular game rule and a particular strategy rule have relatively high scores, the variance of the score in that game rule will increase and the evaluation of the game rule will decrease. Accordingly, the game rules in which many strategy rules have similar scores will have their weight share increase. While one might think this type of "egalitarian" game becomes dominant and an egalitarian society will emerge, any game rule has a difference in scores from the strategy rules unless all the strategy rules are equivalent. Because of the characteristics of replicator dynamics (i.e., strategy rules with higher-than-average scores increase and those with lower-than-average scores decrease their population share), the disparities in the score distribution are

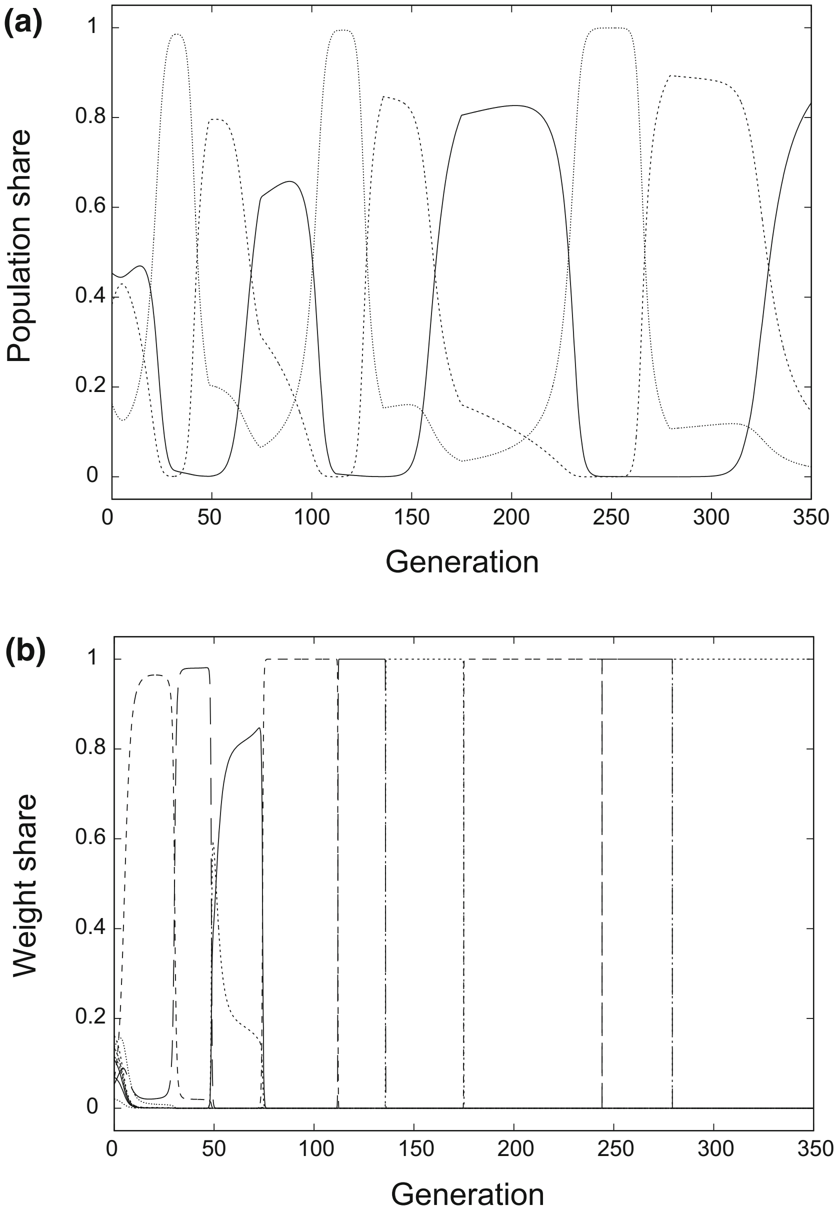

Fig. 2 Typical example of the dynamics of the strategy rules (**a**) and game rules (**b**) within the inverse-variance-type meta-rule (three strategy rules, 10 game rules, $\tau = 1000$)

magnified. When the disparity expands, the variance in that game rule increases and its evaluation decreases. The weight of the game rules with small variances for the strategy rule distribution at that point will increase. This phenomenon will then repeat. In this manner, both types of replicators, game rules (external rules) and strategy rules (internal rules), form cycles under the inverse-variance-type meta-rule. The cycle of the strategy rules precedes that of the game rules. While periods of both cycles are more or less the same, the game rules change more dramatically.

4 Implications

4.1 Rule ecosystem as interactions between the population of strategy rule replicators (internal rules) and that of game rule replicators (external rules as outer institutions)

RED can be written in the form of a multi-population replicator system. A replicator system with two replicator populations, x and y, is generally written as the following system of equations:

$$\dot{x}_i = [u(x_i; y) - \bar{u}(x_i; y)]x_i, \quad (26)$$

$$\dot{y}_i = [u(y_i; x) - \bar{u}(y_i; x)]y_i, \quad (27)$$

where $u(x_i; y)$ is the payoff gained by the i-th individual in population x in competition with population y, and $\bar{u}(x_i; y)$ is the average payoff in population x. Likewise, $u(y_i; x)$ is the payoff gained by the i-th individual in population y in competition with population x, and $\bar{u}(y_i; x)$ is the average payoff in population y. In other words, competition as a game occurs between these two populations, and the results of this competition are compared with those of other individuals in each population. Individuals with a higher-than-average payoff increase their share, whereas those with a lower-than-average payoff decrease their share.

Now, let us consider the case of RED. First, we define matrix $\boldsymbol{A}$ whose $(i_1 \cdots i_M, g)$ element is $(\boldsymbol{A}^g \boldsymbol{x})_{i_1 \cdots i_M}$, which represents payoff $u^g_{i_1 \cdots i_M}$ gained by strategy rule $x_{i_1 \cdots i_M}$ in elemental game rule g, where matrix $\boldsymbol{A}^g$ expresses elemental game rule g. By introducing the vector expression of the weights (the weight profile) $\boldsymbol{w} = (w^1, \cdots, w^g, \cdots, w^M)$, the payoff of strategy rule $x_{i_1 \dots i_M}$ taking account of weight is $(\boldsymbol{A}\boldsymbol{w})_{i_1 \dots i_M}$ and the average within population x is $\boldsymbol{x} \cdot \mathbf{A}\boldsymbol{w}$. As a result, Eqs. (19) and (21) can be rewritten as follows[8]:

$$\dot{x}_{i_1 \cdots i_M} = [(\boldsymbol{A}\boldsymbol{w})_{i_1 \cdots i_M} - \boldsymbol{x} \cdot \boldsymbol{A}\boldsymbol{w}]x_{i_1 \cdots i_M}, \quad (28)$$

$$\tau \dot{w}_g = [\lambda^g(\boldsymbol{x}, \boldsymbol{u}^g) - \bar{\lambda}(\boldsymbol{x}, \boldsymbol{u}^g)]w^g. \quad (29)$$

These equations indicate that RED comprises multi-population replicator dynamics with the interactions between a population of the strategy rules (internal rules) and a population of the game rules (external rules as outer institutions). In other words, each strategy rule interacts with the population of the game rules and competes within the population of the strategy rules, and each game rule interacts

[8] Equation (29) is not represented in matrix form as Eq. (28) because the meta-rule is generally not limited to a linear function.

with the population of the strategy rules and competes within the population of the game rules. This interaction causes the frequency and weight of each to change. In this manner, RED is a model that describes co-evolutionary processes as the interactions between the replicators of the game rules (external rules as outer institutions) at the meso level and the replicators of the strategy rules (internal rules) at the micro level via the vehicles of replicators (i.e., individual cognition, thought, and behavior). In these co-evolutionary processes, both types of replicators not only rise and fall by altering their relative frequencies but also form mutually complementary or alternative relationships.

4.2 Institutional ecosystem from the RED viewpoint

How are institutions understood from the viewpoint of RED? We define institutions as a group of replicators (internal or external rules) stably shared among a relatively large number of individuals (interactors) including organizations (Nishibe 2006, 2010a).

The phrase “stably shared among a relatively large number of individuals (interactors)” within RED means that the relative frequencies of the strategy rules and relative weights of the game rules exceed a certain value for a certain period. The value of the relative frequencies (weights) can be 0.5, 0.1, or 0.01, depending on interest. The issue of how reasonable these values are is reduced to a matter of degree, becoming the point at which we can call these replicators “institutions” on the basis of their dissemination. For example, if we say there is only one institution, the value will be greater than 0.5. If we are interested in understanding the conditions in which many institutions co-exist, the value will be smaller, perhaps 0.1. Furthermore, when we are observing the formation of a new institution, the value may be set to 0.01. We wish to understand the evolution of economic society as an institutional ecosystem wherein institutions are formed, persisted, altered, and extinguished and where multiple institutions co-exist. In this case, we must consider a broad range of relative frequencies for replicators (rules). The period they share is also reduced to a matter of degree and needs to be set appropriately depending on the problem under consideration.

RED can be regarded as a system of the interactions between two types of replicators, strategy rules (internal rules) and game rules (external rules as outer institutions), as described in the previous subsection. Within institutional ecosystem dynamics, game rules such as laws, rules, and regulations become “outer institutions” when they are shared among a relatively large number of individuals. Further, strategy rules such as customs, practices, and values become “inner institutions” when they are shared among a relatively large number of individuals. Just as there are two types of replicators, there are also two kinds of institutions, which interact with each other within the institutional ecosystem, mutually supporting and inducing changes in one another. Traditional views of institutions have focused exclusively on outer institutions and ignored the existence and roles of inner institutions in an attempt to understand institutions. However, within the institutional ecosystem, understanding the interaction dynamics between outer and inner institutions is critical.

What is an institutional ecosystem, where multiple game rules (outer institutions) exhibit complementarity and substitutability relationships and change as they interact along with individual strategies? One example can be found in the Argentine monetary system in the first decade of this century. The Argentine government at the time had defaulted on its foreign debt; the peso, the national currency, subsequently devalued. Multiple currencies were being used in parallel: the key currency (the US dollar), the national currency (peso), bond currencies (patacón and LECOP), and community currencies (such as the RGT) (Gómez 2009). These various currencies were used in different domains, differing in the class they were used, what they could purchase, and in which markets they were used.

By viewing multiple currencies as trying to expand their domain of use, we consider this to be a monetary ecosystem with two parties interacting with each other: currencies as outer institutions (game rules) and the portfolio mixes of these currencies as inner institutions (strategy rules). In the monetary ecosystem, users were vehicles both for currencies as replicators representing the outer institutions and for replicators representing the inner institutions (internal rules) such as the value consciousness toward currency. Both aimed to get as many people using that currency as possible. Users viewed currencies as resources and wished to get as many different types of currencies as possible. At the same time, users contributed to the formation of meta-rules by evaluating the institutions of currencies. This forms the interaction of the replicator dynamics at the two levels in the institutional ecosystem, namely game rules and strategy rules, as shown in Eqs. (28) and (29).

4.3 Meta-rules in the monetary institutional ecosystem

In a monetary institutional ecosystem as seen in the Argentina example, the region and amount of currency distribution change along with users' strategy rules (internal rules); further, the monetary institutional ecosystem itself, with its multiple co-existing currencies, changes, too. The internal rules corresponding herein to the strategy rules include the habits, idiosyncrasies, routines, norms, and values that determine an individual's cognition, thought, and behavior. We believe that people's value consciousness is at the root of the evolutionary processes that change both outer institutions as the game rules (e.g., currency) and inner institutions (e.g., currency portfolios). Value consciousness plays the role of evaluation rules for monetary institutions (i.e., meta-rules in RED). Each individual evaluates the monetary institutions (game rules) by deciding the weight of each currency to be used. Participation in a game is decided by referring to one's internal rules (strategy rules) and payoff, including not only the currency income but also the social and cultural values that one might gain owing to these internal rules. Meta-rules are the basis for determining those weights, but what are they really?

To help answer this question, a money consciousness survey was conducted (Kobayashi et al. 2010, 2013). This survey included 164 subjects consisting of the operators, issuers, and participants of community currencies as well as those who were or had been employed in financial organizations and others (e.g., working people in other sectors, undergraduate and graduate students). They were asked to respond to 27 randomly ordered question items on money consciousness by selecting the most suitable option from five options (strongly agree, weakly agree, neutral, weakly disagree, strongly disagree). A factor analysis of the results suggested three factors: "currency diversity", "fairness", and "profit orientation" (Kobayashi et al. 2010). Such question items as "Do you think it is good that we have different moneys from national currencies to live with?" (Q. 22), "Do you think it is good that money can be created or issued freely by people?" (Q. 4), and "Do you think it is good that money can be issued or created not only by the central bank or commercial banks but also by people or communities?" (Q. 25) loaded highly onto "currency diversity". Such question items as "Do you think the government should provide every adult beyond a certain age with basic income for their minimum standard of living?" (Q. 10), "Do you think it is good that we accommodate each other with money?" (Q. 19), and "Do you think moneylenders should not be in such financial institutions as commercial banks, but the government?" (Q. 13) with respect to the fair distribution of money loaded highly onto "fairness." Such question items as "Do you think it is good for money to be able to buy anything you want?" (Q. 7), "Do you think it is better to earn more money?" (Q. 15), and "Do you think it is good that money can be created or issued for the purpose of profit?" (Q. 12) loaded highly onto "profit orientation."

Considering the relationship with the meta-rules exemplified in Sect. 3, we think that "fairness", which highly evaluates a fair distribution, corresponds to "the inverse-variance-type meta-rule", which aims toward the equality of results. This disposition is in contrast to "profit orientation", which favors having more money for one's convenience, which corresponds to "the average-payoff-type meta-rule." This intends to increase average income, an idea similar to a doctrine of economic growth.

Returning to the results of the simulation analysis above, if everyone had the "profit orientation" meta-rule, then monetary institutions would converge on a single currency that could be used to earn money and spent to buy anything. This prediction implies that currencies, as a platform institution, would be locked in one currency as a de facto standard, which is similar to that forecasted in the argument for standardization competition based on increasing returns and network externalities (Arthur 1994). Even if a monetary institution that aimed for "fairness" (i.e., equality of results) was established, economic activities using that currency would always result in income disparities. Our simulation results thus indicate that the monetary institution would continue to be revised in an effort to achieve true fairness.

How would "currency diversity", the first factor in the money consciousness survey (Kobayashi et al. 2010), be formulated as a meta-rule (game evaluation function) within the RED framework? If we think of "diversity" as the range of score distribution, then we can take a function of variance (a payoff-variance-type meta-rule) thus:

$$\lambda_V^g(\boldsymbol{x}, \boldsymbol{u}) = \sum_{i_1 \cdots i_M} x_{i_1 \cdots i_M} (u^g_{i_1 \cdots i_M} - \bar{u}^g)^2. \tag{30}$$

This meta-rule evaluates the variety in the payoff profile of each strategy rule as a result of interactions. However, question items in the money consciousness survey dealing with "currency diversity" included "Do you think it is good that we have different moneys from national currencies to live with?" (Q. 22), "Do you think it is good that money can be created or issued freely by people?" (Q. 4), and "Do you think it is good that money can be issued or created not only by the central bank or commercial banks but also by people or communities?" (Q. 25). This fact indicates that the co-existence of multiple monetary institutions is considered to be significant. In other words, the diversity of opportunity, not of interaction results, is evaluated. Accordingly, the payoff-variance-type meta-rule that evaluates the diversity of payoff profiles can only be used as an indirect result (i.e., the 0th order approximation) of the diversity of the game rules (outer institutions).

Unfortunately, the diversity of the game rules cannot be directly evaluated by using the RED formulated thus far. Therefore, we now try to extend the RED framework. To do so, Eq. (21) needs to be redefined to include a weight profile, $\boldsymbol{w}$, into the meta-rule evaluation function:

$$\lambda^g = \lambda^g(\boldsymbol{x}, \boldsymbol{u}^g, \boldsymbol{w}). \tag{31}$$

Then, we introduce a function to determine the evaluation of the game rules (i.e., a meta-rule) by considering their diversity with the weight variance or distance of weights from the average. We cannot use the weight variance solely to evaluate the diversity of the game rules because, as seen in the following equation:

$$\lambda^g_{\mathrm{WV}}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{w}) = \sum_{g=1}^{M} (w^g - \bar{w}^g)^2, \tag{32}$$

which is determined by the weight profile of all the game rules rather than used to evaluate each game rule. This meta-rule is not an evaluation of each monetary institution, but rather a basis for assessing the state of the overall monetary ecosystem (or the diversity of the institutions in an ecosystem). Therefore, we consider the evaluation to be the distance from the largest weight, denoted as w^{MAX}:

$$\lambda^g_{\mathrm{WD}}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{w}) = w^{\mathrm{MAX}} - w^g. \tag{33}$$

This meta-rule highly evaluates the game rules with small weights. In other words, it works to evaluate the game rules that have not been evaluated. Accordingly, we expect the weights to become homogenized over time.

Alternatively, we can also suppose a meta-rule that evaluates the closeness of the weight of each game rule to the average weight, instead of the overall variance of the weight profile:

$$\lambda^g_{\mathrm{WD}}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{w}) = (w^g - \bar{w}^g)^{-2}. \tag{34}$$

In addition, the fact that three meta-rules were found through the money consciousness survey suggests that all three co-existed simultaneously. The allocations of those meta-rules likely varied by person. Indeed, Kobayashi et al.

(2010) showed the different emphases of people related to community currencies and people working in the financial sector. Therefore, we must pursue our analysis by combining the multiple meta-rules in RED as below:

$$\lambda^g(\boldsymbol{x}, \boldsymbol{u}) = \alpha\lambda^g_{\mathrm{A}} + \beta\lambda^g_{\mathrm{IV}} + \gamma\lambda^g_{\mathrm{WD}} \quad (\alpha + \beta + \gamma = 1). \tag{35}$$

These meta-rules are a combination of functions with different behavior such as the variance- and the inverse-variance-types, and their dynamics are likely to be complex.

4.4 RED as micro–meso–macro loops

A dynamic society can be viewed as a micro–meso–macro loop: institutions mediate between individuals' cognition, thought, and behavior (micro) and societal consequences (macro) at the meso level, and they are formed, maintained, altered, and diminished through the interactions among individuals. Substitutive and complementary relationships among institutions are formed in that process. The micro, with the meso as an intermediary, determines the macro and, at the same time, the macro, with the meso as an intermediary, determines the micro (Nishibe 2006). Let us reconsider RED from this perspective.

First, we schematize the replicator system from this perspective (see Fig. 3). The strategy rules (internal rules) of individuals at the micro level interact through the game rules (external rules as outer institutions). A payoff distribution is then formed through these interactions as a societal consequence at the macro level. Replicator dynamics determine the change in the distribution of the strategy rules in response to the payoff distribution, and a regulative relationship from macro to micro is formed. The game rules are thought to be at a dimension that mediates between the micro and macro (i.e., the meso level). If the game rules (game forms) are viewed as external rules, we can make the following contrasts: micro: meso: macro = strategy rules (internal rules): game rules (external rules as outer institutions): societal consequences. In usual replicator dynamics, however, there are no regulatory relationships from macro to meso; thus, there are no changes in the game. This makes it impossible to describe and analyze the dynamics of institutions, namely the formation and alterations through micro–meso–macro loops.

Within arguments that consider the equilibria of strategic forecasting as institutions (Lewis 1969; Pagano 1992; Pagano and Rowthorn 1994; Young 1998; Aoki 2001), a strategy distribution at an equilibrium becomes an institution when positioned at the meso level as a shared belief (Fig. 4). From this standpoint, micro, meso, and macro are all maintained statically, and regulatory relationships from macro to micro work to stabilize and self-maintain the institution.

Next, we consider the micro–meso–macro loops within a rule ecosystem (Fig. 5). Because we have introduced meta-rules herein, the states of the micro and the macro influence the game rules via the meta-rules. Thus, two regulatory relationships are added: one is a relationship from the macro to meta-rules at the meso, while the other is that from the micro to the meta-rules. Both these relationships influence the

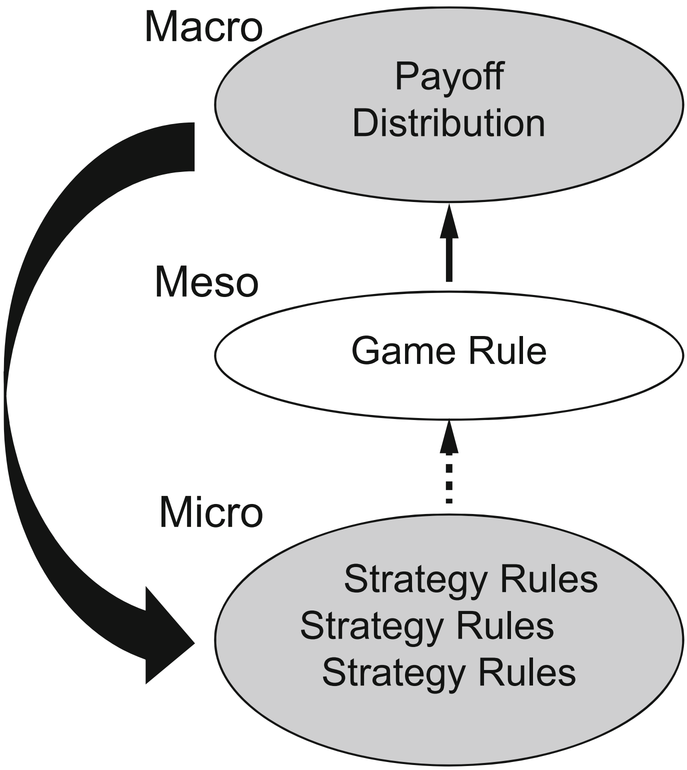

Fig. 3 Micro–meso–macro loop in a replicator system. The *dotted line* represents inputs, while the *black lines* show effects. *Gray areas* are dynamic, while the *white area* is static

evaluation of the game rules through the meta-rules. As a result, loops among micro–meso–macro levels are formed, and the game rules change dynamically.

That the framework of the rule ecosystem is a replicator system with time-varying games is more easily understood by the following formulation. We characterize the state of a society consisting of individuals playing multiple games with the weighted average of elemental games. We introduce $\boldsymbol{G}_{\mathrm{T}}$ to represent this, called the total game:

$$\boldsymbol{G}_{\mathrm{T}} = \sum_{g=1}^{M} w^g \boldsymbol{A}^g, \tag{36}$$

where $\boldsymbol{A}^g (g = 1, \ldots, M)$ is the payoff matrix of elemental game g. As clearly shown by $w^g = w^g(t)$, which changes over time, $\boldsymbol{G}_{\mathrm{T}}$ is a function that depends on time. The dynamics of the strategy rules distribution in RED (19) can be shown in matrix form by using the total game:

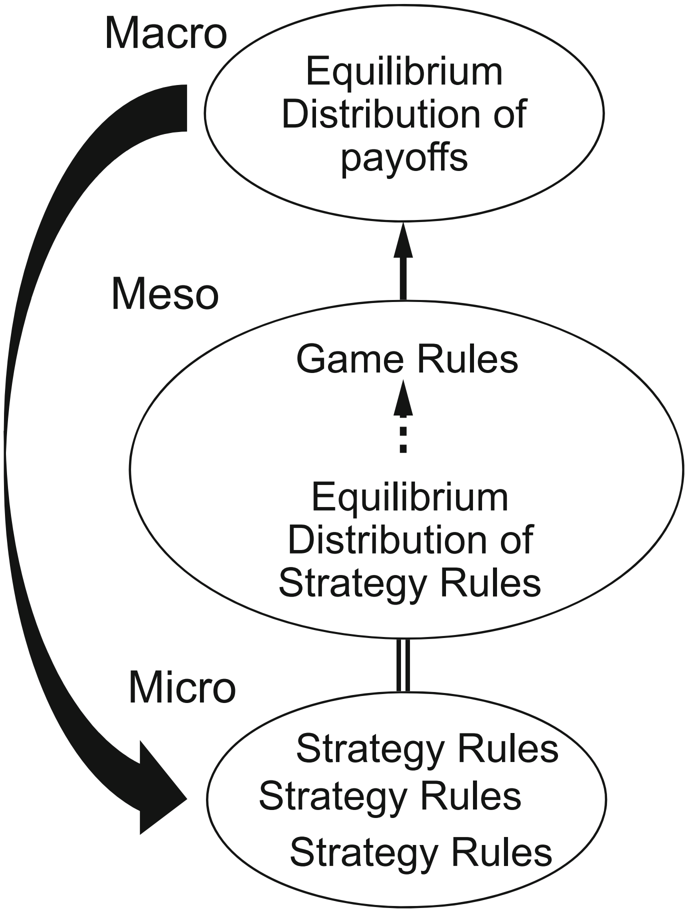

Fig. 4 Micro–meso–macro loop of a replicator system at an equilibrium state. The *dotted line* represents inputs, while the *black lines* show effects. The *double line* means equivalent. The *white area* is static

$$\dot{x} = (\boldsymbol{G}_{\mathrm{T}}(t) - \boldsymbol{x} \cdot \boldsymbol{G}_{\mathrm{T}}(t))\boldsymbol{x}. \tag{37}$$

In other words, this system is an extended replicator equation that has two layers of sets of replicators, a set of the strategy rules and that of the game rules, which interact and develop over time through an interaction matrix $\boldsymbol{G}_{\mathrm{T}}(t)$ that also changes over time.

As with the equilibrium distribution of the strategy rules, we can also consider an equilibrium of the game rules. Under strategy rule profile $\boldsymbol{x}$ and meta-rule λ^g, when game rule g (or a total game) satisfies

$$\lambda^g(\boldsymbol{x}, \boldsymbol{u}^g) > \lambda^{g'}(\boldsymbol{x}, \boldsymbol{u}^{g'}) \tag{38}$$

for any game rule g', game rule g is stable in response to invasion by any game rule. Further, if strategy rule profile $\boldsymbol{x}$ is an evolutionary stable strategy rule (ESSR)

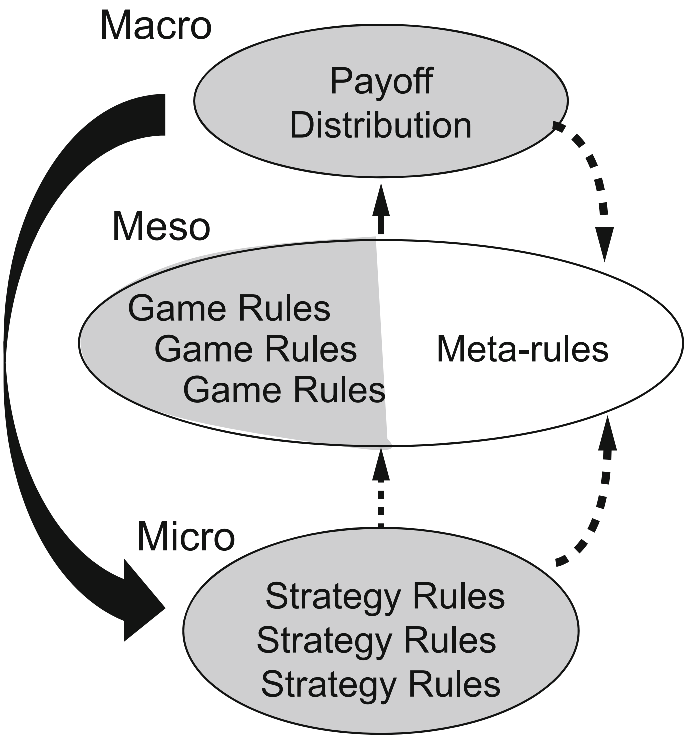

Fig. 5 Micro–meso–macro loop of a rule ecosystem. The *dotted lines* represent inputs, while the *black lines* show effects. *Gray areas* are dynamic. The *white area* is static

profile, then there is no incentive for either the game rules or the strategy rules to change. Thus, that system will be in a state of equilibrium for both types of rules. We call game rule g under ESSR profile $\boldsymbol{x}$ evolutionary stable game rules (ESGRs).

When both ESGRs and ESSRs exist, the overall rule ecosystem is in an equilibrium and the micro, meso, and macro will not change; Fig. 6 presents the micro–meso–macro loop. As with the average-payoff-type meta-rule, simple meta-rules also manifest in this type of equilibrium state. Further, there are meta-rules such as the inverse-variance-type meta-rule in which the dominant game rules are constantly invaded by the other game rules. As noted previously, in the real world, it is typical for these types of meta-rules to be combined and seldom for ESGRs such as $\lambda^{g}(\boldsymbol{x}, \boldsymbol{u}^{g}) > \lambda^{g'}(\boldsymbol{x}, \boldsymbol{u}^{g'})$ for all games g' to exist.

5 Conclusion

In this paper, we introduced RED as an integration and extension of evolutionary game theory and replicator dynamics and observed the realization of dynamic changes in both the strategy rules and the game rules. By treating both strategies and games in a unified manner as rules, two game-theoretic viewpoints of institutions

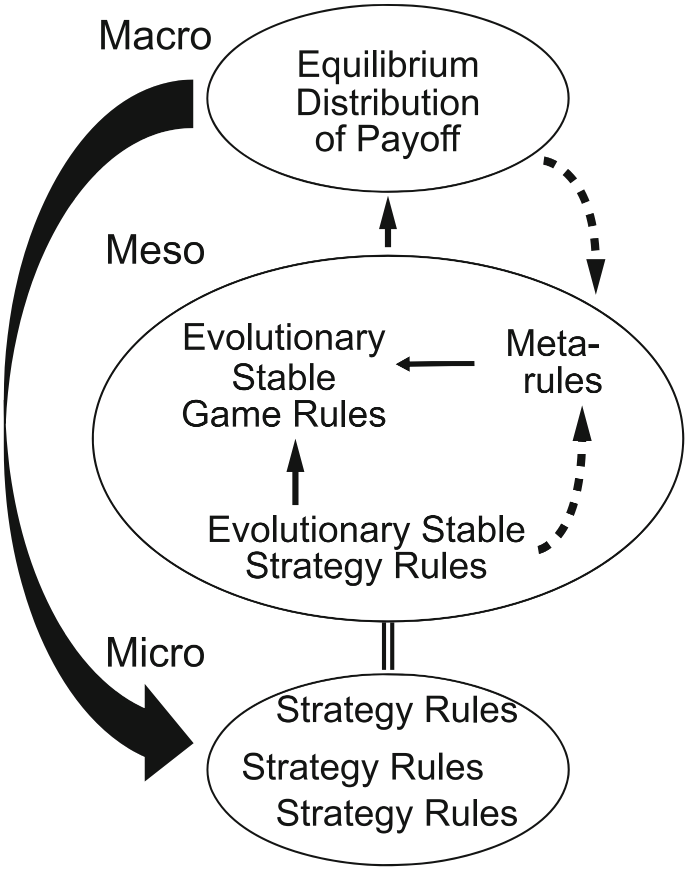

Fig. 6 Micro–meso–macro loop of a rule ecosystem at an equilibrium state with ESSRs and ESGRs. The *dotted lines* indicate inputs, while the *black lines* show effects. The *double line* means equivalent. The *white areas* are static

can be successfully integrated. We also observed intermittent changes among the quasi-stable states of the strategy rules and game rules, as in the case of the inverse-variance-type meta-rule.

Because we can view the rule ecosystem as a multi-population replicator system, the game rules may also be considered to be replicators, or external rules, that are replicated through interactions with individuals' strategy rules. To deal with the duality of replicators (i.e., game rules and shared strategy rules), or internal rules, the concept of the micro–meso–macro loop is necessary. Here, we introduce the meso level, which connects the micro and macro levels.

By discussing the monetary institutional ecosystem and results from a money consciousness survey, we formulate a meta-rule that expresses "diversity" as the primary consciousness for money. To directly evaluate the diversity of institutions, we examine an extension of the RED framework, finding that an analysis of systems that integrate three meta-rules, namely diversity, fairness, and profit orientation, is necessary. Analyzing the dynamics of such a system is, therefore, a topic for future research.

Meta-rules are expressed collectively as individuals' weighted game evaluations and are difficult to manipulate. If we do not assume a priori rationality according to neoclassical economics, we should consider micro-level replicators, or strategy rules, as internal rules that are determined by individual cognitive frameworks, practices, routines, and values. These become cultures, traditions, norms, and societal consciousness (money consciousness), or inner institutions, when they are shared. Working on these internal rules and inner institutions consciously to change is not necessarily impossible but it becomes a pressing policy matter. For example, Japan implemented a policy of promoting energy conservation by appealing to people's sense of danger in response to a nuclear accident. However, it is, of course, difficult to directly manipulate and design people's sense for this purpose. Evolutionist institutional design has been proposed to indirectly influence these replicators through the institutional design of games, or external rules, as outer institutions, notably through the design of platform institutions such as currencies and accounting systems (Nishibe 2006, 2010b). Further developing and substantiating the RED framework would thus enable us to make simulation models that could be used in evolutionist institutional design.

Acknowledgements This research was partly supported by JSPS Kakenhi Nos. 17651088 and 21330063.

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